

Thermodynamic analysis of onset characteristics in a miniature thermoacoustic Stirling engine

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This paper analyzes the onset characteristics of a miniature thermoacoustic Stirling heat engine using the thermodynamic analysis method. The governing equations of components are reduced from the basic thermodynamic relations and the linear thermoacoustic theory. By solving the governing equation group numerically, the oscillation frequencies and onset temperatures are obtained. The dependences of the kinds of working gas, the length of resonator tube, the diameter of resonator tube, on the oscillation frequency are calculated. Meanwhile, the influences of hydraulic radius and mean pressure on the onset temperature for different working gas are also presented. The calculation results indicate that there exists an optimal dimensionless hydraulic radius to obtain the lowest onset temperature, whose value lies in the range of 0.30–0.35 for different working gases. Furthermore, the amplitude and phase relationship of pressures and volume flows are analyzed in the time-domain. Some experiments have been performed to validate the calculations. The calculation results agree well with the experimental values. Finally, an error analysis is made, giving the reasons that cause the errors of theoretical calculations.

Keywords: Stirling thermoacoustic engine; high frequency; thermodynamic; onset temperature

Introduction

Thermoacoustic engine (TE) converts heat into acoustic power and has the advantage of no moving parts and thus promises a long life. The TEs have been applied in some field such as cryogenic refrigeration and electric generation. In standing-wave TE [1], the energy conversion between the gas and solid through irreversible thermal contacts and the theoretical thermal efficiency is less than 20%. The thermoacoustic Stirling heat engine (TASHE), firstly proposed by Backhaus and Swift [2], employs an inherently reversible thermodynamic circle and the thermal efficiency can reach up to 30%. The linear thermoacoustic theory [3] is successful in explaining

the conversion from thermal energy to acoustic energy. The DeltaEc, developed by Ward and Swift based on linear thermoacoustic theory, is a powerful tool for optimizing and designing TEs. However, some nonlinear phenomenon, such as onset characteristics and mode transition, are still not fully understood. So far, few research works involve the onset characteristics of TE. Lai [4] studied the onset temperature of a standing wave TE based on the thermoacoustic network theory, however, the method cannot be applied directly to the TASHE for the absence of practical geometrical closed end in the loop; Benavides [5] predicted the onset temperature by taking the output work of gas circle to be zero as the critical onset conditions, however, the coupling of oper-

ating frequency and onset temperature has not been taken into account in his model; Hu [6] proposed the two-port network model and obtained the startup criteria for TE by using Nyquist instability criterion. Recently, de Waele [7] proposed a thermodynamic model to reveal the onset conditions in TASHEs, focusing on the onset temperature and oscillation frequency. In this model, some assumptions have been made for simplicity. The regenerator is treated to be ideal (zero void volume), which indicates the compliance effect of regenerator is neglected. Meanwhile, the acoustic field in the regenerator is assumed to be a pure traveling wave, meaning the oscillating pressure and volume flow are in phase. In an actual TASHE, however, the acoustic field in the regenerator is neither a pure standing wave nor a pure traveling wave but a traveling-standing wave [8]. The performance of TASHE, including the onset characteristics, is strongly dependent on the acoustic field. The working conditions (such as working gas or mean pressure) and regenerator's geometrical parameters (such as porosity or hydraulic radius) have great influence on the acoustic field. Therefore, a more practical thermodynamic model needs to be developed. In this paper, a similar model is built with a more realistic treatment of regenerator based on de Waele's research. Numerical calculations and experiments are carried out to investigate the onset characteristics of a miniature TASHE.

Governing equations

The miniature TASHE is schematically shown in Fig. 1(a). The main part of the miniature TASHE consists of cold heat exchanger (CHX), regenerator, heating block, pulse tube and ambient heat exchanger (AHX); together with feedback tube, inertance tube and 180° bend cavity, they form a closed loop. The resonator connects to the loop with a three way tube. The dimensions of components in the loop are much smaller than the wavelength of oscillating working gas, so they can be modeled as lumped elements. The void volumes of CHX, AHX and heating block are small enough to be negligible. The inertance tube is modeled as a solid piston with a mass M_i equal to the mass of working gas contained in the inertance tube; only the compliance effect is considered in the feedback tube, 180° bend tube, pulse tube and only the inertance effect is considered in the resonator. The resonator is modelled as a cylinder with volume V_R (average length L_R), closed by a resonating piston with mass M_R . The value of M_R is equal to the mass of the gas contained in the resonator in front of the piston. Let the basic angular resonance frequency be equal to the resonator frequency of the mass-gas spring system [2, 8], it has $L_R = 2L_{ac}/\pi$. In this way, the TASHE can be simplified to a thermodynamic model as shown in Fig. 1(b).

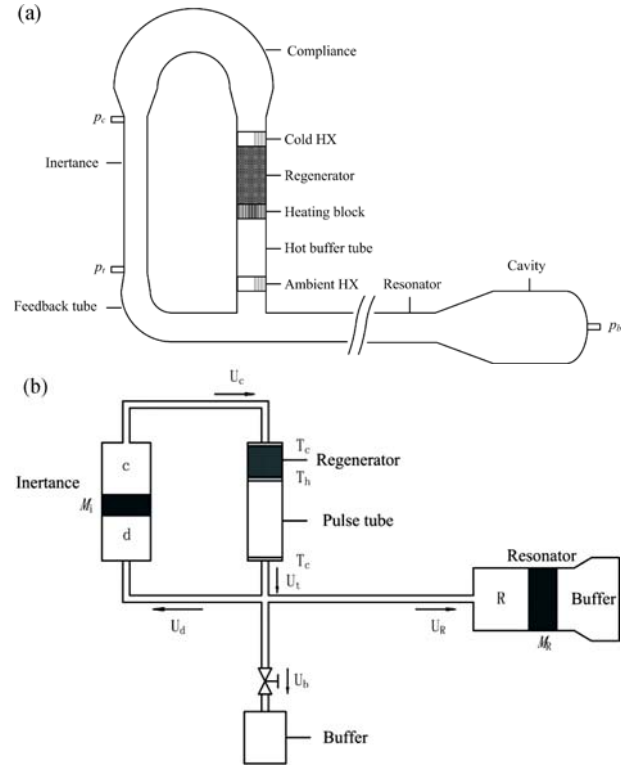


Fig. 1 Schematic drawing of the miniature TASHE: (a) before thermodynamic analogy conversion; (b) after thermodynamic analogy conversion.

Table 1 Main dimensions of the miniature TASHE

System parameter	Sym	Value
Diameter of resonator	D_R	0.016 m
Length of resonator	L_{ac}	0.35 m
Volume of cavity	V_b	$5.4 \times 10^{-4} \text{ m}^3$
Diameter of regenerator	D_r	0.021 m
Length of regenerator	L_r	0.02 m
Length of pulse tube	L_t	0.06 m
Diameter of pulse tube	D_t	0.022 m
Volume of space d	V_{d0}	$3.84 \times 10^{-5} \text{ m}^3$
Length of inertance tube	L_i	0.1245 m
Diameter of inertance tube	D_i	0.022 m
Volume of space c	V_{c0}	$8.5 \times 10^{-5} \text{ m}^3$
Ambient temperature	T_c	300 K

The usual complex notation for the time-oscillating pressure in the system is:

$$p = p_m + \delta p = p_m + \text{Re}[\hat{p}e^{i\omega t}] \quad (1)$$

where p_m is the mean pressure, δp is the time-dependent oscillating pressure, which is given by the real part of the product of complex amplitude $\hat{p}$ and $e^{i\omega t}$. Other variables (such as volume flow rate U and volume V) have the same form as the oscillating pressure p . It is assumed

that the time-dependent oscillating variables are much smaller than the mean variables. In the following analysis, we will focus on the time-dependent oscillating variables and all the dynamic equations are in time-domain.

The equation of volume d is given by:

$$U_d = \frac{dV_d}{dt} + \frac{V_{d0}}{\gamma p_0} \frac{dp_t}{dt} \quad (2)$$

The dynamic equation of mass of the inertance tube is:

$$\frac{dV_d^2}{dt^2} = \frac{A_i^2}{M_i} (p_t - p_c) \quad (3)$$

The volume flow entering into volume c is described by:

$$U_c = \frac{dV_d}{dt} - \frac{V_{c0}}{\gamma p_0} \frac{dp_c}{dt} \quad (4)$$

The equation of pulse tube is:

$$U_h = U_t + \frac{V_t}{\gamma p_0} \frac{dp_t}{dt} \quad (5)$$

The length of regenerator is much shorter than the wavelength of working gas in the system. The momentum equation and continuity equation of the regenerator in frequency-domain are [3]:

$$\hat{p}_t - \hat{p}_c = F_v \hat{U}_c \quad (6)$$

$$\hat{U}_h - \hat{U}_c = F_k \hat{p}_c + F_g \hat{U}_c \quad (7)$$

$$F_v = -\frac{i\omega\rho_m L_r}{1 - f_v A_g} \quad (8)$$

$$F_k = -\frac{i\omega V_r}{\gamma p_m} [1 + (\gamma - 1) f_k] \quad (9)$$

$$F_g = \frac{(\tau - 1)(f_k - f_v)}{(1 - f_v)(1 - \sigma)} \quad (10)$$

where L_r , A_r , V_r are the length, gas area of cross section, volume of the regenerator; σ , γ are Prandtl number and specific heat ratio; τ is the dimensionless temperature, which is defined as the ratio of hot end temperature T_h and ambient end temperature T_c of regenerator; f_v , f_k are the viscous and thermal functions related to geometry and gas property. For ease of calculation, the randomly oriented stainless-steel screens are modeled by a bundle of parallel circular pores with the same hydraulic radius r_h as that of the screens. In this case, the specific expressions of f_v , f_k are written as [3]:

$$f_v = \frac{2J_1[(i-1)2r_h/\delta_v]}{J_0[(i-1)2r_h/\delta_v] + [(i-1)2r_h/\delta_v]} \quad (11)$$

$$f_k = \frac{2J_1[(i-1)2r_h/\delta_k]}{J_0[(i-1)2r_h/\delta_k] + [(i-1)2r_h/\delta_k]} \quad (12)$$

$$\delta_v = \sqrt{2\mu/\omega\rho_m} \quad (13)$$

$$\delta_k = \sqrt{2k/\omega\rho_m c_p} \quad (14)$$

where δ_v , δ_k are the viscous penetration depth and thermal

penetration depth; μ , k are dynamic viscosity and thermal conductivity of the gas; c_p is the specific heat per unit mass at constant pressure.

Writing equations (6)-(7) in the time-domain, we have:

$$p_t - p_c = \chi_v \left(1 + \lambda_v \frac{\partial}{\partial t}\right) U_c \quad (15)$$

$$U_h - U_c = \chi_k \left(1 + \lambda_k \frac{\partial}{\partial t}\right) p_c + \chi_g \left(1 + \lambda_g \frac{\partial}{\partial t}\right) U_c \quad (16)$$

where χ , λ are the proportional coefficient and first-order differential coefficient, respectively. They can be calculated by using the following complex formulation:

$$\chi_i (1 + i\omega\lambda_i) = F_i (i = v, k, g) \quad (17)$$

Mass conservation at the T type junction gives:

$$U_t = U_d + U_b + U_R \quad (18)$$

The acoustic power dissipation outside the regenerator (the major part of dissipation is in the resonator) is modeled by an orifice and a buffer volume (which is assumed to be very large). The volume flow rate through the orifice is U_b :

$$U_b = C_o \delta p_t = C' \text{Re}(f_{k_R})^2 \delta p_t \quad (19)$$

where f_{k_R} is the thermal function in the resonator and can be calculated according to equation (12).

The volume flow in the resonator is given by:

$$U_R = \frac{dV_R}{dt} + \frac{V_R}{\gamma p_0} \frac{dp_t}{dt} \quad (20)$$

The equation of the motion of the mass M_R in the resonator is:

$$\frac{d^2 V_R}{dt^2} = \frac{A_R^2}{M_R} (\delta p_t - \delta p_b) \quad (21)$$

The pressure variation in the buffer is assumed to be an adiabatic process and then it gives:

$$\frac{V_b}{\gamma p_0} \frac{dp_b}{dt} = \frac{dV_R}{dt} \quad (22)$$

Equations (2)-(5), (15)-(16), (18)-(22) form an equation set, from which the onset temperature and oscillation frequency can be known. The equation set is closed with eleven variables which are p_t , p_c , p_b , U_d , U_c , U_h , U_t , U_b , U_R , V_d and V_R . It's difficult to drive an analytic solution, so we resort to numerical calculation.

Calculation and discussion

In this part, some calculation results will be presented according to the above relations. The main dimensions of TASHE are listed in Table 1. The regenerator is made of randomly stacked screens with a porosity of 0.815 and a hydraulic radius of 0.25 mm. If not stated otherwise, the working gas is helium with mean pressure 2.0 MPa and the length and diameter of resonator tube are 0.35 m and

0.016 m respectively.

Oscillation frequency

For the TASHE system, oscillation frequency is an important parameter which is mainly determined by the working gas and geometrical dimensions of the components. Table 2 lists the oscillation frequencies of four different working gases. Calculating results of DeltaEc model [9] and simple lumped parameters method [2, 10] are also listed for comparison. It is found that the DeltaEc model predicts the frequencies with a quite high precision, which shows good agreement with the experimental results. The thermodynamic model provides a lower precision than the DeltaEc model but a higher accuracy than the simple lumped parameters method.

The effects of resonator length on the oscillation frequency are shown in Fig. 2. The difference between the thermodynamic model and DeltaEc calculation frequency decreases as the length of resonator increases. When the length of resonator increases from 0.2 m to 0.6 m, compared with the experiment results, the relative error decreases from 41.2% to 17.7%.

Table 2 Oscillation frequencies of different working gases.

	He	N ₂	Ar	CO ₂
DeltaEc	245.8	84.0	76.3	67.2
Thermodynamic	330.0	110.0	103.9	84.4
Simple lumped	417.3	147.6	135.4	112.7
Experimental	245.0	83.4	75.5	66.7
Deviation (%)	30.3	27.5	29.9	26.5

Fig. 3 gives the frequency as a function of resonator diameter. In small-scale TASHE, the influence of the resonance tube diameter on the operating frequency is quite sensitive. The increasing of resonator diameter will lead to a remarkable increase in frequency. However, the deviations between the DeltaEc model and thermodynamic model do not change much with resonator diameter.

Onset temperature

Onset temperature is a key parameter for evaluating the performance of TE. Low onset temperature has a vital significance on the utilization of low-grade energy [11]. The onset temperature of TE is influenced greatly by the radial dimension of regenerator, which is characterized by the hydraulic radius r_h . A large r_h means irreversible thermal contacts between the working gas and the stuff in the regenerator. If r_h is big enough, the thermoacoustic effect will be negligibly small.

In this case, the TE will begin self-excited oscillation at an extremely high temperature or even fail to self-oscillate.

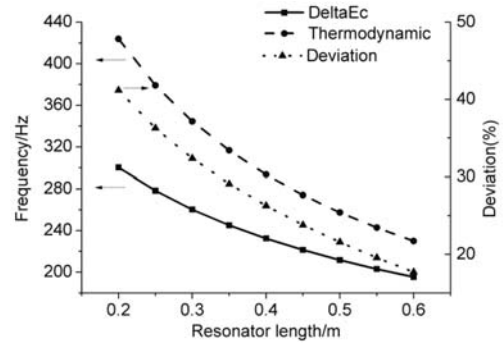


Fig. 2 Oscillation frequency as a function of resonator length

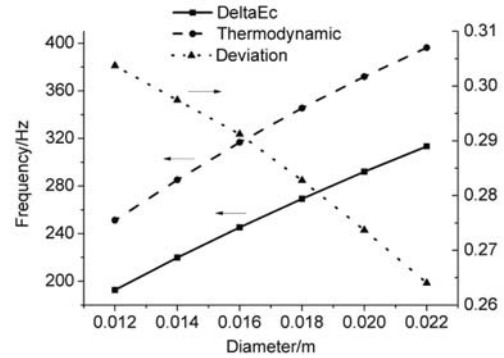


Fig. 3 Oscillation frequency as a function of resonator diameter

However, the decrease of r_h will lead to a small flow resistance of the regenerator and thus increase the viscous dissipation of acoustic power. Therefore, a too small r_h will also cause a high onset temperature. There must be a compromise between the irreversible losses and the viscous dissipation of the acoustic power. Then, there exists an optimum value of r_h for the TE to obtain lowest onset temperature.

Fig. 4 shows the calculated dimensionless onset temperature as a function of hydraulic radius of the regenerator. The dimensionless onset temperature is defined as the ratio of temperature at the hot end of regenerator to that at the warm end ($T_c=300$ K). Obviously, there exists an optimal hydraulic radius corresponding to the lowest onset temperature. On the left of minimum onset temperature, onset temperature increases abruptly with the decrease of hydraulic radius, while at the right side, the onset temperature rises with a smaller rate. The optimal hydraulic radius corresponding to the lowest onset temperature increases with the mean pressure. Here, the thermal penetration depth is used to nondimensionalize the hydraulic radius. The dimensionless hydraulic radius D_h is defined as the ratio of hydraulic radius to thermal penetration depth:

$$D_h = \frac{r_h}{\delta_k} \quad (23)$$

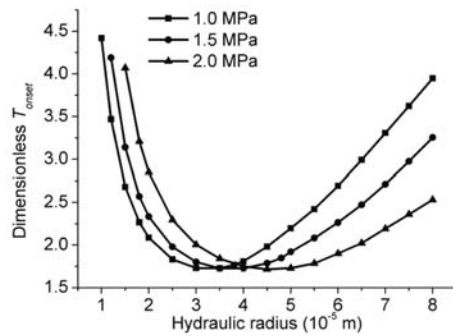


Fig. 4 Dimensionless onset temperature as a function of hydraulic radius

Equations (14) and (23) indicate that the dimensionless hydraulic radius varies with the mean pressure as well as the hydraulic radius of regenerator. Therefore, we can investigate the effect of dimensionless hydraulic radius on the onset temperature by varying the mean pressure. Fig. 5(a) gives the calculated dimensionless onset temperatures for four different working gases as a function of mean pressure. There is an optimal mean pressure for the TASHE to obtain the lowest onset temperature for each working gas. With the mean pressure increasing from 0.5 MPa to 3.0 MPa, the onset temperature of helium (He) decreases monotonically and reaches the lowest dimensionless onset temperature of 1.7 at 3.0 MPa; the mean pressures corresponding to the lowest temperature of nitrogen (N_2) and argon (Ar) both are 1.2 MPa; the optimal mean pressure of carbon dioxide (CO_2) is 0.8 MPa. When the mean pressure is less than 1.5 MPa, the onset temperature of CO_2 is the lowest among the four working gases; however, helium has the lowest temperature when the mean pressure is higher than 2.5 MPa. The onset temperatures of Ar are always higher than N_2 from 0.5 MPa to 3.0 MPa.

The influence of mean pressure on the onset temperature can be attributed to their effect on the dimensionless hydraulic radius. By calculating the dimensionless hydraulic radius of regenerator with different mean pressures, we get the effects of dimensionless hydraulic radius on the onset temperature as shown in Fig. 6(a). Obviously, the optimal dimensionless hydraulic radiuses for the lowest onset temperature of each working gas are nearly the same and the values are about 0.30-0.35.

Pressure and Volume flow

Proper amplitude and phase relationship of the oscillating pressures and volume flows have a great significance for the TASHE to obtain a good performance. Due to the resistance, inertial and compliance effects of the working gas, the pressures and volume flows in each component are quite different. In the thermodynamic model, there are three pressure variables and six volume flow variables as shown in Fig. 1(b).

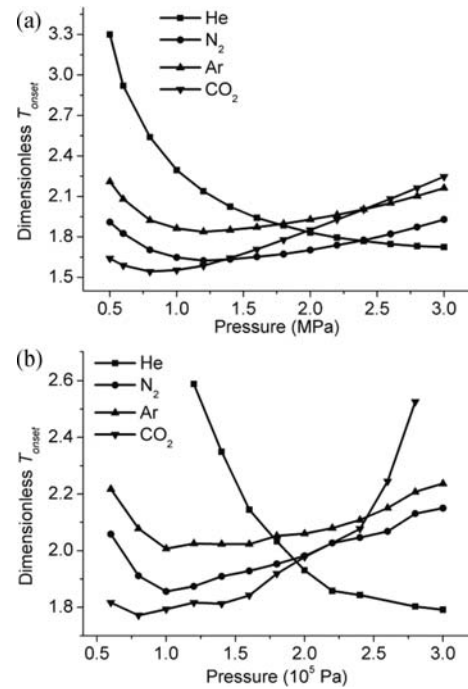


Fig. 5 Dimensionless onset temperature as a function of mean pressure: (a) in calculations; (b) in experiments.

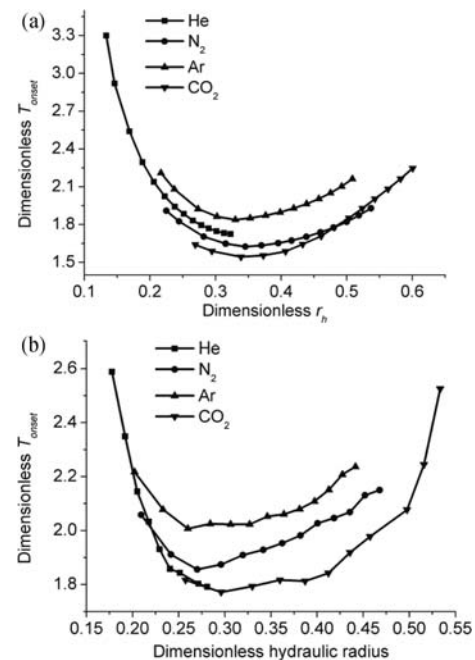


Fig. 6 Dimensionless onset temperature as a function of dimensionless hydraulic radius: (a) in calculations; (b) in experiments.

Fig. 7 illustrates the pressure fluctuations in the time-domain, from which the amplitude and phase relationship are clearly presented. In the calculations, the pressure amplitude at the cold end of regenerator p_c is set as 0.135 MPa, which is a typical value from experiments. The

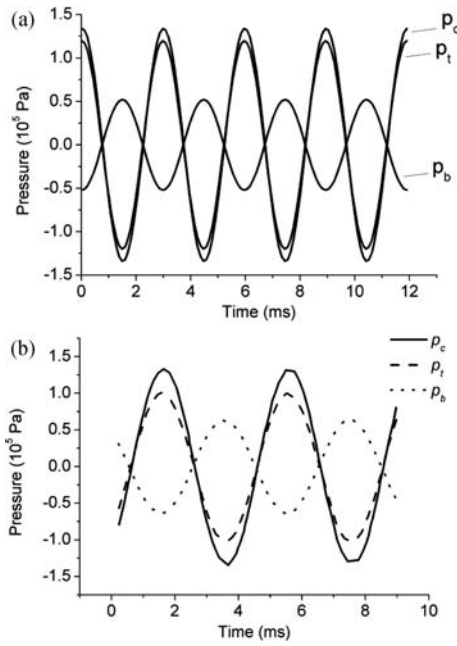


Fig. 7 Pressure fluctuations in the time-domain: (a) in calculations; (b) in experiments.

pressure amplitude at the hot end of regenerator p_h is 0.12 MPa, which has a decline comparing to p_c due to the flow resistance effect of the regenerator.

For the negligible inertial effect of regenerator, p_c and p_h are nearly in phase. The pressure oscillation at the cavity is much weaker than at the regenerator for the compliance effect of cavity. The amplitude of p_b is 0.52 MPa and the phase difference between p_c and p_b is about 180°.

Fig. 8 shows the volume flow fluctuations in the time-domain. The volume flow at the hot end of pulse tube U_t is split into three parts at the T type junction. The left one U_d is fed back to the inertance tube; the right one U_R flows into the resonator and the third part U_o passes through the orifice, representing the viscous loss in the resonator. The relationship of the volume flows at the three-way tube is represented by equation (18). The phase difference of U_d and U_R is about 180°, meaning that the

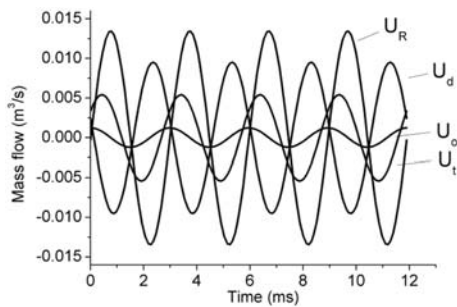


Fig. 8 Volume flow fluctuations in the time-domain.

two flows are in opposite phase. The amplitude of U_o is relatively small compared to the other three volume flows.

Fig. 9 shows the phase diagram of the pressures and flow volumes at the two ends of the regenerator. The phase difference at the cold end of the regenerator is 22.3°, with p_c leading U_c ; and increases to 35.9° at the hot end due to the compliance effect of regenerator on volume flow. The phase relationships calculated above are well matched with the DeltaEc calculation results [9]. The acoustic power gain, which is defined as the difference between the acoustic power at the hot end of regenerator and that at the cold end, is often used to evaluating the performance of the regenerator.

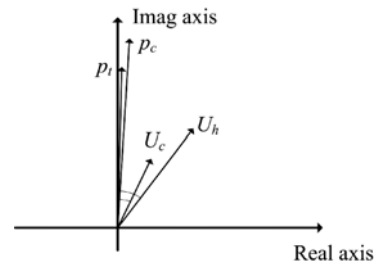


Fig. 9 Phase diagram of pressure and flow volume at the two ends of regenerator. A quantitative scale is omitted.

The acoustic power flowing can be written as $E=1/2|p||U|\cos\phi$, therefore the acoustic power at the two ends of regenerator is 135.9 W and 205.1 W, respectively. So, the acoustic power gain is 69.2 W, which is a little higher than that of the DeltaEc result of 60.0 W.

Experiments and verification

Experiments have been performed to validate the calculation results. A detailed introduction to the experimental apparatus can be found in reference [9, 10]. The heating power input is fixed at 150 W. Other working conditions are the same as in the calculations which have been introduced in part 3. As shown in Fig. 1(a), pressure sensors labeled P_c , P_t and P_b are located at the two ends of the inertance tube and the end of the buffer cavity, in order to compare with the three calculation pressure variables. The pressure sensors are of the piezoelectric type (CY-YD 208), supplied by Sinocera Piezotronics, Inc, China, with the total measuring error of ± 750 Pa. A NiCr-NiSi thermocouples is placed in the heating block to measure the temperature (with accuracy $\pm 1^\circ$).

The frequencies for different working gases are measured as presented in Table 2. The deviations between the thermodynamic model calculation and experimental frequencies are about 30%. In essence, the thermodynamic model is built up based on the lumped-parameter as-

sumption that most components of the TASHE are equated to resistance, inertance or compliance.

The treatments of these components are not accurate enough, especially for the small-scale TASHE in the paper. It has been validated indirectly that the deviation of frequency between the calculation and experiment decrease as the length of resonator increases as shown in Fig. 2. This means the thermodynamic model has higher accuracy for larger TASHE system. The experimental values of onset temperature for different working gases are also presented in Fig. 5(b), 6(b). As shown in these figures, the calculated curves can reflect the trends of the experimental data. The measured onset temperature is higher than the computed value. The reason may lie in the difference between the model and the actual TASHE. The pressure waves are also measured as shown in Fig. 7(b) and show good agreement with the calculated oscillation pressures.

Conclusions

The onset characteristics of a miniature TASHE were studied based on the thermodynamic model method, focusing on the oscillation frequency and onset temperature. The calculation results indicate that the thermodynamic model can make rather remarkably accurate predictions about the varied trends of frequency according to working gas and resonator dimensions. The onset temperatures of four working gases with different mean pressures are also calculated. Though the optimal mean pressures for each working gas are different, the corresponding dimensionless hydraulic radii are indeed almost the same, whose values lie in the range of 0.30–0.35. The amplitude and phase relationship of the oscillating pressures and volume flows are presented in the time-domain. The phase difference in the regenerator is close to the traveling-wave angle (0°), which is beneficial for the TASHE to obtain a good performance. Meanwhile, experiments have been performed to validate the calculations. The frequencies and onset temperatures are measured and they are in good agreement with the calculations.

Acknowledgement

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References

- [1] G.W. Swift, Analysis and performance of a large thermoacoustic engine. *J. Acoust. Soc. Am.* 92(3) (1992) 1551–1563.
- [2] S. Backhaus, G.W. Swift, A thermoacoustic Stirling heat engine: detailed study. *J. Acoust. Soc. Am.* 107 (2000) 3148–3166.
- [3] G.W. Swift, *Thermoacoustics: A Unifying Perspective for Some Engines and Refrigerators*. ASA, Melville, NY, 2002.
- [4] B.H. Lai, L.M. Qiu, Y.F. Li, Simulation of onset process of standing wave thermoacoustic engine based on thermoacoustic network theory. *J. Zhejiang Univ. (Engineering Science)* 45(6) (2011) 1130–1135 (in Chinese).
- [5] E.M. Benavides, An analytical model of self-starting thermoacoustic engines. *J. Appl. Phys.* 99(11) (2006) 114905.
- [6] X.H. Hu, X.Q. Zhang, H.L. Wang, S.M. Shu, Two-port network model and startup criteria for thermoacoustic oscillators. *Chin. Sci. Bull.* 54(2) (2009) 335–343.
- [7] A.T.A.M. de Waele, Basic treatment of onset conditions and transient effects in thermoacoustic Stirling engines. *J. Sound. Vib.* 325 (2009) 974–988.
- [8] H.F. Kang, Q. Li, G. Zhou, Optimizing hydraulic radius and acoustic field of the thermoacoustic engine. *Cryogenics* 49 (2009) 112–119.
- [9] G. Zhou, Q. Li, Z.Y. Li, A miniature thermoacoustic Stirling engine. *Energ. Conv. Manage.* 49 (2008) 1785–1792.
- [10] G. Zhou, Q. Li, Z.Y. Li, Influence of resonator diameter on a miniature thermoacoustic Stirling heat engine. *Chin. Sci. Bull.* 53(1) (2008) 145–154.
- [11] Z.B. Yu, A.J. Jaworski, Optimization of thermoacoustic stacks for low onset temperature engines. *Proc. IMechE, Part A: J. Power.* 224(2010) 329–337